Nuclear Criticality Safety Engineer Training
Module 1 [1]

Introductory Nuclear Criticality Physics

LESSON OBJECTIVES

    1) to introduce some background concepts to engineers and scientists who do not have an educational background in nuclear engineering, including the basic ideas of moles, atom densities, cross sections and nuclear energy release;

    2) to discuss the concepts and mechanics of nuclear fission and the definitions of fissile and fissionable nuclides.

NUCLEAR CRITICALITY SAFETY

The American National Standard for Nuclear Criticality Safety in Operations with Fissionable Materials Outside Reactors, ANSI/ANS-8.1 includes the following definition:

    Nuclear Criticality Safety: Protection against the consequences of an inadvertent nuclear chain reaction, preferably by prevention of the reaction.

Note the words:

    nuclear - related to the atomic nucleus;
    criticality - can it be controlled, will it run by itself;
    safety - protection of life and property.

DEFINITIONS AND NUMBERS

What is *energy*?	Energy is the ability to do work.
What is *nuclear energy*?	Energy produced by a nuclear reaction.
What is *work*?	Work is force times distance.

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    [1] Developed for the U. S. Department of Energy Nuclear Criticality Safety Program by T. G. Williamson, Ph.D., Westinghouse Safety Management Solutions, Inc., in conjunction with the DOE Criticality Safety Support Group.

Push a car (force) along a road (distance) and the car has energy of motion, or kinetic energy. Climb (force) a flight of steps (distance) and you have energy of position relative to the first step, or potential energy. Jump down the stairs or out of a window and the potential energy is changed to kinetic energy as you fall. When you hit the ground the kinetic energy is converted to destructive energy of broken bones, etc.

A common unit of energy in nuclear physics is the electron volt, eV.

> An *electron volt* is the increase in kinetic energy of an electron when it falls through an electrical potential of one volt.

One electron volt is equivalent to

$$1 \text{ eV} = 1.602 \text{ X } 10^{-19} \text{ joules},$$

where a *joule* is the metric unit of energy - a force of one newton acting through one meter.

The energy release from the fission of a ^{235}U or ^{239}Pu nucleus is about 200 MeV (million electron volts). Is that a large or a small amount of energy? Obviously the answer depends on what is meant by large and small. How much would a 200 MeV "hotfoot" change the potential energy of a 150-pound man; i.e., how high would he jump? Or, how many steps would a person have to climb to gain a potential energy equivalent to 200 MeV? The calculation is simple:

$$E = \text{ m x g x h} = 200 \text{ MeV}$$

With m = 150 lb x 0.454 kg/lb and g = 9.8 m/s^2, the answer is

$$h = 5 \text{ x } 10^{-14} \text{ m}.$$

An atomic radius is about 10^{-10} m or 10^{-8} cm, and a nuclear radius is about 10^{-14} m or 10^{-12} cm. So the victim of this "hotfoot" only jumps about a nuclear diameter, not even so far as from a good hiccough.

As a comparison to the average fission energy, the *atomic binding energy*, the energy needed to remove an electron from a heavy atom, is about 10 - 100 keV. The *nuclear binding energy*, the energy to remove a neutron or proton from a nucleus, is about 8 MeV.

Thus the energy of fission, 200 MeV, wreaks havoc on an atomic or nuclear scale but doesn't do much on an everyday scale of climbing steps and jumping out of windows. If nuclear energy is so small on an everyday scale why do we worry about it? The answer is in the product of large and small numbers.

Avogadro's Number is defined as:

$$N_a = 6.0221 \times 10^{23} = 0.60221 \times 10^{24}$$

which is the number of atoms (or molecules) in one *mole* of any substance.

(**Note**: N_a is given to five significant figures, since it has been carefully determined over the long history of its use. Generally, the values of parameters important to criticality safety such as atom densities and computed multiplication factors may not be known to this precision and care must be exercised to treat these values with the proper level of significance. Often more than the significant number of digits are carried through a calculation, then truncated to the proper level to avoid compounding round-off errors.)

A mole of a substance is that amount of a substance having a mass in grams equal to the atomic or molecular weight of that substance. The *atomic weight* of an atom is defined as the mass of a neutral atom relative to the mass of a neutral ^{12}C atom, on a scale in which the atomic weight of ^{12}C is taken to be exactly 12.0. Thus twelve grams of ^{12}C constitutes one mole of ^{12}C.

The atomic weight of the uranium isotope ^{235}U, A_{U-235}, is 235.0439. That is, one mole of ^{235}U weighs 235.0439 grams. Then the number of atoms per gram of ^{235}U is

$$
\begin{aligned}
N &= N_a \text{ (atoms/mole) } / A_{U-235} \text{ (grams/mole)} \\
&= 6.0221 \times 10^{23}/235.0439 \\
&= 2.5621 \times 10^{21} \text{ atoms/g.}
\end{aligned}
$$

Now a "hotfoot" caused by the energy equivalent to that released by the fission of 1 g of ^{235}U raises the 150-pound man (using the result of the 200 MeV calculation)

$$
\begin{aligned}
h \times N &= 5 \times 10^{-14} \text{ m} \times 2.5621 \times 10^{21} \text{ atoms} \\
&= 1.3 \times 10^{8} \text{ m,}
\end{aligned}
$$

and he is certainly now out of this world.

A few other numbers: the *mass of a ^{235}U atom*

$$
\begin{aligned}
M_{U-235} &= 1/N \text{ (atoms/g)} \\
&= 3.9 \times 10^{-22} \text{ g/atom.}
\end{aligned}
$$

(**Note**: there are several common conventions used to denote specific isotopes of an element, using superscripts and subscripts. Among physicists, the preferred notation is:

$$_p^a X_n$$

where X represents the nuclide, p is the atomic number (number of protons in the nucleus), n is the number of neutrons, and a is the total number of nucleons (protons plus neutrons). Among chemists, the preferred notations is:

$$_p X_n^a$$

The two notations are often used interchangeably, and often several of the numbers are omitted, but it is usually clear what isotope is intended by the author.)

The area of a nucleus is approximately $(10^{-12} \text{ cm})^2 = 10^{-24} \text{ cm}^2$. This area is a commonly adopted unit for the cross section of the nucleus, and is given a special name, the *barn*.

$$1 \text{ barn } = 10^{-24} \text{ cm}^2$$

Within a given material, the *atom densities* of a given nuclide, N_i, can be calculated from:

$$N_i = \rho \times (\text{weight fraction})_i \times N_a / A_i$$

where ρ is the density of the material and (weight fraction)$_i$ is the weight fraction of nuclide i in the material. If N_a is given the value of 0.60221, this expression will compute the atom densities with units of atoms/barn-cm. This is a convenient unit because atom densities in these units can be multiplied by cross sections with units of barns to get interaction rates without having to carry large or small exponents.

As an example, consider plutonium metal composed of 95 wt-% ^{239}Pu and 5 wt-% ^{240}Pu. The metal density is 19.6 g/cm^3 and the isotopic weights of ^{239}Pu and ^{240}Pu are 239.0522 and 240.0538, respectively. Then

$$N_{\text{Pu-239}} = 19.6 \times 0.95 \times 0.60221 / 239.0522$$

$$= 4.6907 \times 10^{-2} \text{ at/barn-cm}$$

and

$$N_{\text{Pu-240}} = 19.6 \times 0.05 \times 0.60221 / 240.053$$

$$= 2.4585 \times 10^{-3} \text{ at/barn-cm}$$

The atomic weight of this plutonium metal is

$$A_{Pu} = \sum_i (\text{atomfraction})_i * A_i$$

or

$$\frac{1}{A_{Pu}} = \sum_i \frac{(\text{wt.fraction})_i}{A_i}$$

giving A_{Pu} = 239.199 g. Note that the difference between weight-fraction and atom-fraction is important when doing criticality safety calculations.

The *microscopic cross section*, σ, can be defined as a representation of the effective area of the nucleus for certain types of collisions with neutrons; it can also be defined in terms of the probability that a given nucleus will interact with a neutron. The total cross section of a material indicates the effectiveness of that material in removing neutrons from an incident beam of neutrons. For an incident beam of neutrons with intensity I_0, the intensity of the beam emerging from a target of thickness x is given by

$$I(x) = I_0 e^{-N\sigma_t x}$$

where σ_t is the microscopic total cross section and N is the atom density of the material. The product $N\sigma$ is called the *macroscopic cross section*, denoted by Σ. Subscripts are added to indicate various cross sections such as scattering (Σ_s) and total (Σ_t). When more than one isotope is present in a material, such as a mixture of plutonium isotopes or a chemical compound, the macroscopic cross sections are additive. For example, given a sample of uranium metal consisting of only ^{235}U and ^{238}U. The macroscopic cross section is given by

$$\Sigma = \Sigma_{235} + \Sigma_{238} = N_{235}\, \sigma_{235} + N_{238}\, \sigma_{238}$$

FISSION

Return once more to the concept of energy and the famous $E = mc^2$ equation. Define an *atomic mass unit* (amu) as a mass with atomic weight A = 1.0000 exactly. (Atomic weights are taken from the General Electric Chart of Nuclides and Isotopes, Fourteenth Edition, 1989, and generally will be written to four places after the decimal.) The mass of one atomic mass unit is

$$1 \text{ amu} = A_{amu} / N_a$$

$$= 1.6606 \text{ x } 10^{-24} \text{ g} \quad (\text{or } 1.6606 \text{ x } 10^{-27} \text{ kg})$$

In energy units,

$$1 \text{ amu } = mc^2$$

$$= 1.6606 \times 10^{-27} \text{ kg x } (2.9979 \times 10^{8} \text{ m/s})^{2}$$

$$= 1.4925 \times 10^{-10} \text{ joules}$$

$$= 931.50 \text{ MeV} .$$

The complete conversion of one atomic mass unit to energy would release about 932 MeV.

The masses of the electron, the neutron and the hydrogen atom are:

Particle	Mass (amu)	Mass (MeV)
$_{-1}^{0}e$	5.4858×10^{-4}	0.5110
$_{0}^{1}n$	1.008664	939.5721
$_{1}^{1}H$	1.007825	938.7906

The difference between the masses of the neutron and the hydrogen atom is 0.782 MeV. This energy is the beta decay energy released when a neutron decays to a proton. (A free neutron decays with a 10.4 minute half-life.)

In the fission process a nucleus, usually a heavy nucleus, splits into two (or occasionally three) fragments or *fission products*. The fragments have a large amount of energy (on a nuclear scale), which is mostly in the form of kinetic energy as the fragments move away from each other. In addition to the fission products, several neutrons with significant kinetic energy are normally liberated, as well as several gamma-ray photons.

One example of a fission reaction is given by the following equation.

$$_{0}^{1}n + \ _{92}^{235}U \rightarrow \ _{38}^{91}Sr + \ _{54}^{143}Xe + 2_{0}^{1}n + E$$
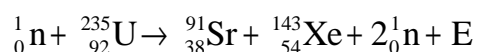
The masses involved in this reaction are shown in the following table. The difference between the sum of the initial masses and the sum of the final masses is 0.1977 amu or 184 MeV, the energy released in fission.

Initial Masses (amu)		Final Masses (amu)	
^{235}U	235.0439	^{91}Sr	90.9102
^{1}n	1.0087	^{143}Xe	142.9273
		2 ^{1}n	2.0174
Total	236.0526	Total	235.8549

The mass of a nucleus is slightly less than the sum of the masses of its constituents (neutrons and protons). The difference is the *mass defect* or *binding energy* of the nucleus, the energy required to break up the nucleus into its separate parts. The concept of the energy released when a heavy nuclide splits approximately in two is illustrated in Figure 1, which is a plot of the binding energy per nucleon against atomic number.
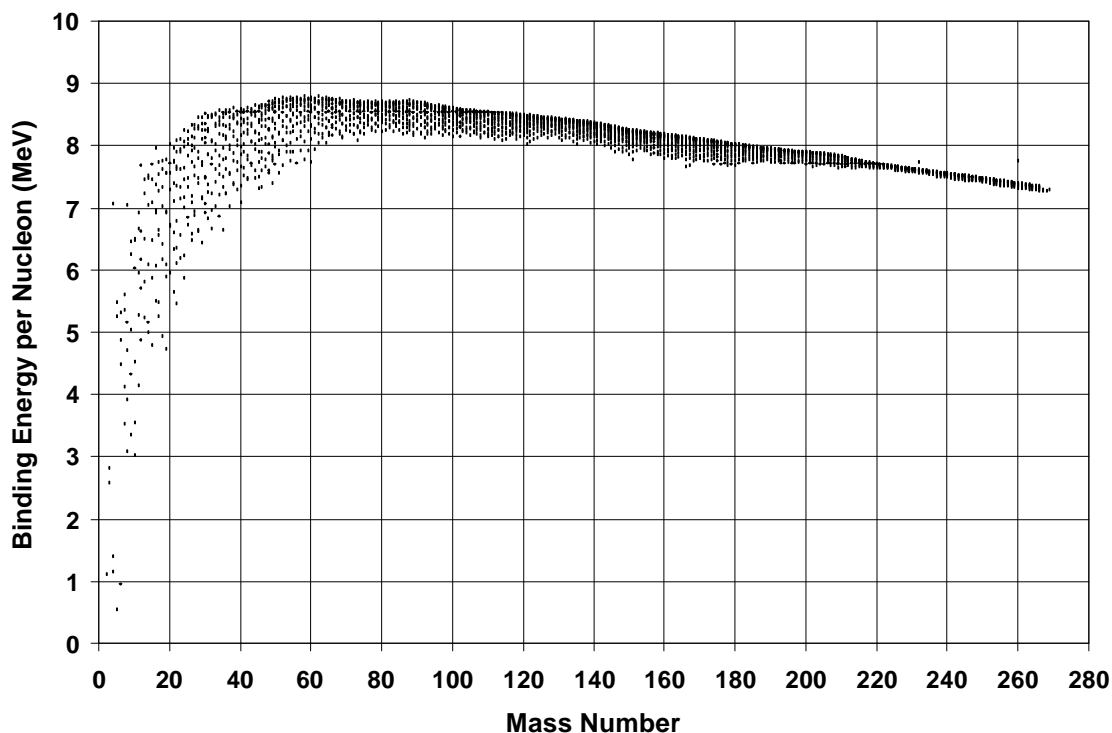
Figure 1. Binding Energy Per Nucleon as a Function of Atomic Mass Number

At an atomic number of 235, the binding energy per nucleon is about 7.6 MeV. At atomic numbers of about half this value, near 120, the binding energy per nucleon is about 8.4 MeV. The difference, 0.8 MeV per nucleon times 235 nucleons is 188 MeV. This value corresponds closely to the value introduced above. Note also from Figure 1 that it requires about 8 MeV to remove a nucleon from most nuclei. At the end the curve of low atomic numbers, the curve is very steep, which means that if two very light isotopes can be combined the energy release per nucleon is large. This is the realm of nuclear fusion.

LIQUID DROP MODEL

The liquid drop model of the nucleus is a convenient model to describe the nuclear fission process. Consider the nucleus as a spherical drop of liquid which is agitated. The spherical nucleus distorts, elongates, approaches a dumbbell shape, and separates into two parts. The nuclear forces which hold the nucleus together are short range forces, with effective ranges on the order of 10^{-14} m. As the nucleus distorts beyond the range of the nuclear forces, the electrostatic repulsive forces between the positive proton charges drive the two fragments apart.

This model gives a fair picture of the energies involved and can be used to calculate the energy needed to induce fission, i.e., the energy required to get the fission products separated. The difference between the initial and the final energies of the system is the Q value of the fission reaction. Then the energy which must be supplied to induce fission, E_{crit}, is

$$E_{crit} = E_q - Q$$

where E_q is the Coulomb (i.e., electrostatic) energy at the point at which the fragments separate. A rough estimate of this energy can be made by assuming that each fragment is a sphere just at separation and that

$$E_q = \frac{Z_1 Z_2 e^2}{R_1 + R_2}$$

where Z and R are the charge and radius of the two fission fragments. The nuclear radii are approximated by

$$R \sim \frac{r_e}{2} A^{1/3}$$

and

$$r_e \sim \frac{e^2}{m_e c^2}$$

where r_e is the classical radius of the electron, m_e is the electron mass and A is the atomic weight of one of the fission fragments.

Then

$$E_q = \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} * 2 m_e c^2$$

or since $2\, m_e c^2 \, . \, 1$ MeV,

$$E_q = \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} \text{ (MeV)} \quad .$$

If we also assume that fission leads to fragments of equal mass and charge then this equation reduces to

$$E_q = 0.16 \frac{Z^2}{A^{1/3}} \text{ (MeV)}$$

For ^{235}U, E_q is approximately 218 MeV and the average Q value is 212 MeV, so the critical energy is about 6 MeV. This is the energy required, on Figure 2, to move the nucleus from the "well" in the nucleus over the hump.
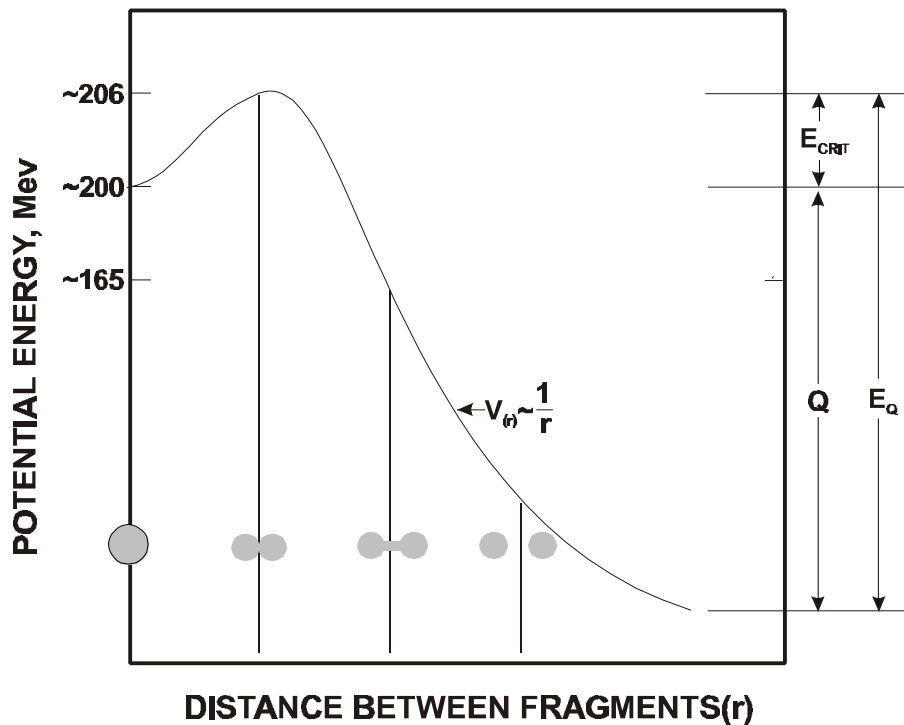
Figure 2. Potential Energy of Nucleus Undergoing Fission

How can this energy be supplied? One way is the binding energy of the "added" neutron. For example, in the reaction

$$_{92}U^{235} + _{0}n^{1} \rightarrow \left(_{92}U^{236} \right)^{*}$$
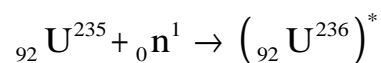
(**Note:** the * indicates an excited or excess-energy configuration) the neutron binding energy, or the difference in masses between the left side and the right side of this equation, is about 6 MeV. (This energy could also be supplied by a 6 MeV gamma ray, which could also induce fission in ^{235}U.)

The following table gives some values for the critical energy and the binding energy of the neutron for several neutron induced fission reactions.

Isotope	Compound Nucleus	Critical Energy (MeV)	Binding Energy of Neutron
^{232}Th	^{233}Th	5.9	5.1
^{235}U	^{236}U	5.75	6.4
^{238}U	^{239}U	5.85	4.9
^{239}Pu	^{240}Pu	5.5	6.4

Those reactions for which the neutron binding energy exceeds the critical energy can be caused by a low (zero) energy neutron. The trend is for the critical energies to decrease with increasing atomic number. For nuclei lighter than uranium the critical energies are considerably higher, for example about 20 MeV for ^{208}Pb. For heavier elements the critical energies are considerably lower and many of the heavier nuclei decay by spontaneous fission with half lives much shorter than those of the uranium isotopes. There is a narrow range of nuclei for which fission occurs by low energy neutrons.

For ^{238}U the reaction

$$_{92}\text{U}^{238} + _{0}\text{n}^{1} \rightarrow \left(_{92}\text{U}^{239} \right)^{*}$$
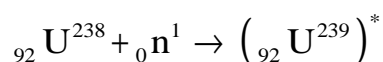
has a Q value of 4.9 MeV. This is less than the critical energy, so approximately one MeV must be supplied (in the center-of-mass coordinate system) to exceed the critical energy and induce fission in ^{238}U. Thus ^{238}U will only fission by neutrons with energy above a threshold energy of about one MeV.

Nuclei are grouped into categories that reflect the numbers of protons and neutrons in each nucleus. These categories are even-even, even-odd, odd-even and odd-odd, where the first word reflects the number of protons and the second word the number of neutrons. The following table illustrates the nucleon makeup of several heavy isotopes.

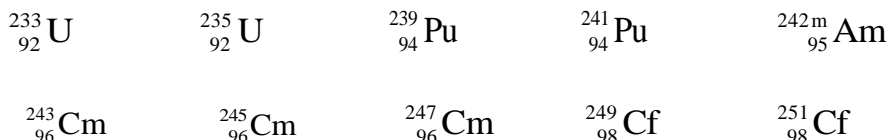| Isotope	Number of Protons	Number of Neutrons	Nucleus Type
^{235}U	92	143	even-odd
^{236}U	92	144	even-even
^{238}U	92	146	even-even
^{236}Np	93	143	odd-odd
^{237}Np	93	144	odd-even
^{239}Pu	94	145	even-odd
^{240}Pu	94	146	even-even
^{241}Pu	94	147	even-odd
^{241m}Am	95	147	odd-odd

The most stable configurations, that is the nuclei with the largest neutron binding energy, occurs when the nuclei are paired, i.e., even-even nuclei. ^{235}U needs a neutron to go to the more tightly bound state of ^{236}U and will accept one readily. ^{238}U does not need a neutron. In uranium and plutonium, each with even numbers of protons, neutron capture to an even-even nucleus should be likely to cause fission. In neptunium and americium, each with an odd number of protons, neutron capture to an odd-even nucleus should be more likely to cause fission than neutron capture to an odd-odd nucleus, although ^{242m}Am is an exception. (Note: the superscript m indicates an excited or "metastable" state within the nucleus.)

FISSILE NUCLIDES

ANSI/ANS-8.15, the American National Standard for Nuclear Criticality Control of Special Actinide Elements, defines a *fissile nuclide* as

> a nuclide capable of undergoing fission by interaction with slow neutrons provided the effective thermal neutron production cross section exceeds the effective thermal neutron absorption cross section.

The fissile nuclides of concern in criticality commonly include:

$^{233}_{92}$U $^{235}_{92}$U $^{239}_{94}$Pu $^{241}_{94}$Pu $^{242m}_{95}$Am

$^{243}_{96}$Cm $^{245}_{96}$Cm $^{247}_{96}$Cm $^{249}_{98}$Cf $^{251}_{98}$Cf

Other nuclides which can be made to fission with neutrons, but not with thermal neutrons, are sometimes referred to as *fissionable nuclides*. This list includes:

$$^{237}_{93}\text{Np} \qquad ^{238}_{94}\text{Pu} \qquad ^{240}_{94}\text{Pu} \qquad ^{242}_{94}\text{Pu} \qquad ^{241}_{95}\text{Am} \qquad ^{243}_{95}\text{Am}$$

Criticality safety characteristics of many of these nuclides are listed in ANSI/ANS-8.15.

It is of some interest to note that ^{238}U is not on these lists, although we have noted that fission can be induced in that nuclide, but only with high energy neutrons. Because of its high threshold energy and other neutron-removing properties, it is normally not a criticality safety concern.

NUMBER OF NEUTRONS FROM FISSION

When a heavy nuclide fissions, typically more than one neutron will be emitted. The number of neutrons is normally given the symbol ν. The value of ν is unique for each nuclide, although generally it is about 2.5 neutrons per fission. The number of prompt neutrons per fission, ν, has a slight dependence on the energy of the neutrons causing fission.

$$\nu(E) = \nu_0 + a*E$$

where the constant a has units of MeV $^{-1}$. Two typical values are:

$$^{239}\text{Pu} \qquad \nu^{49} = 2.874 + 0.138 * E$$

$$^{235}\text{U} \qquad \nu^{25} = 2.432 + 0.066 * E \qquad \text{for } (0 < E < 1)$$
$$= 2.349 + 0.15 * E \qquad \text{for } (E > 1)$$

Note that the value of ν for ^{239}Pu is about 20% higher than that for ^{235}U, one factor which makes plutonium more reactive than uranium.

FISSION NEUTRON ENERGY SPECTRUM

Neutrons from fission are "fast" neutrons. Later we will define "fast" more carefully, but for now it is enough to say that on the average a neutron from a fission reaction will have an energy of about 2 MeV. The neutrons are actually emitted with a spectrum of energies from about 100 keV to about 20 MeV.

A common notation for the energy distribution of neutrons from fission is $\chi(E)$, with the normalization that

$$\int_0^\infty \chi(E)dE = 1$$

A simple fit to the distribution is given by the Maxwellian distribution

$$\chi(E) = \frac{2}{\sqrt{\pi}} T^{-3/2} E^{1/2} e^{-E/T}$$

where the single parameter T is the nuclear temperature. This parameter is related to the average fission neutron energy by

$$T = \frac{2}{3}\overline{E} \quad .$$

The following table gives some values of T for various isotopes to illustrate the point that different fissile isotopes have different characteristic spectra.

Isotope	T (MeV)	Type of Fission
^{233}U	1.306	thermal
^{235}U	1.290	thermal
^{239}Pu	1.333	thermal
^{252}Cf	1.466	spontaneous
^{244}Cm	1.33	spontaneous
^{240}Pu	1.27	spontaneous

For thermal fission of ^{235}U the distribution has been better fit with the following widely used expression

$$\chi(E) = 0.453e^{-1.036E} \sinh\sqrt{2.29E}$$

The average energy of this distribution is 1.98 MeV and the most probable energy is 0.78 MeV. This fission distribution, which has been shown to match experimental data, is shown in Figure 3. In this figure the scales on the bottom and on the left go with the top curve, and the scales on the top and on the right go with the lower curve.
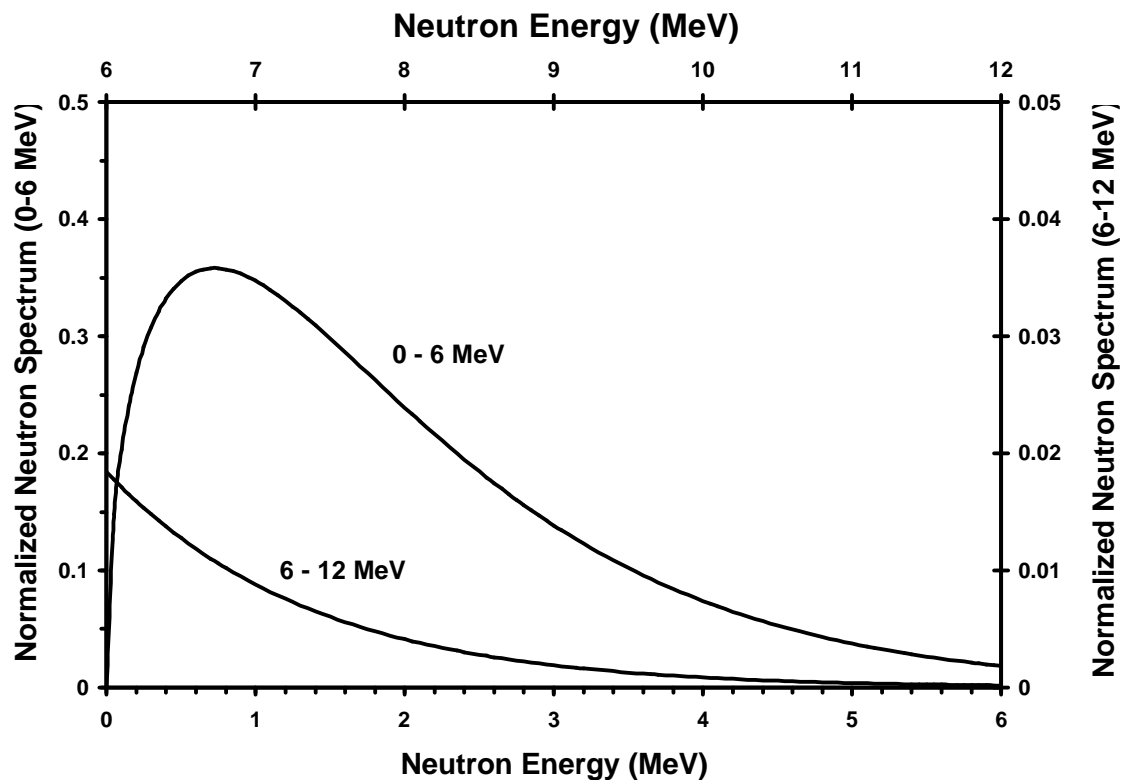
Figure 3. Spectrum of Neutrons from Fission of ^{235}U.

SUMMARY

In this lesson the basic concepts of nuclear criticality physics were introduced including the definitions of

       energy and the mass equivalent of energy,

       Avogadro's number, moles, and the atomic weight of isotopes,

       the size of the atomic nucleus

       cross sections

       binding energy, nuclear fission and the energy released by a fission event, and

       fissile and fissionable nuclides, neutron spectra.

PROBLEMS

1. Plutonium metal with a density of 19.6 g/cm^3 has the following isotopic composition (in weight percent): ^{239}Pu, 90.5 wt-%, ^{240}Pu, 5.7 wt-%, ^{241}Pu, 2.3 wt-% and ^{242}Pu, 1.5 wt-%. Calculate the atom density for each isotope in atoms/barn-cm. Calculate the atom fraction of each isotope and calculate the atomic weight of this plutonium mixture.

2. Calculate the neutron binding energy of the product nucleus for the reactions ^{A}Pu + n 6 $^{A+1}$ Pu for neutron capture in the plutonium isotopes ^{238}Pu, ^{239}Pu, ^{240}Pu and ^{241}Pu.

3. The radius of an atom is often approximated by the expression

$$R = 1.25 \times 10^{-13} \, A^{1/3} \text{ cm}$$

where A is the atomic mass number of the nucleus. For hydrogen, oxygen and ^{235}U, calculate the nuclear radius, the cross sectional area, the volume and the density of the nuclear matter.

4. Compute the atom densities in atoms/barn-cm of ^{235}U, ^{238}U and O in UO$_2$ with a density of 10.8 g/cm^3 if the uranium is enriched to 93 wt-% ^{235}U.

5. Stainless steel 304 has a density of 7.86 g/cm^3. The nominal composition by weight and the thermal neutron cross sections (in barns) of the components are:

	wt-%	σ_a	σ_s
Fe	70.92	2.56	11.35
C	0.08	0.0035	4.74
Cr	19	3.07	3.38
Ni	10	4.49	17.8

Calculate the atom densities (in atoms/barn-cm) of each constituent of SS-304. Calculate the macroscopic scattering, absorption and total cross sections (in cm^{-1}) for the steel.

PROBLEM SOLUTIONS

1. From Page 4, the atom density is

$$N_i = \rho \times (\text{weight fraction})_i \times N_a / A_i$$

Using $A_{239} = 239.0522$, $A_{240} = 240.0538$, $A_{241} = 241.0568$ and $A_{242} = 242.0587$, the atom densities in atoms/barn-cm are:

$N_{239} = 4.4685 \times 10^{-2}$ $\qquad\qquad$ $N_{240} = 2.8027 \times 10^{-3}$
$N_{241} = 1.1262 \times 10^{-3}$ $\qquad\qquad$ $N_{242} = 7.3143 \times 10^{-4}$

The atom fraction of each isotope is simply the ratio of each atom density to the total atom density. The sum of the above values is 4.9345×10^{-2}, giving fractions of

$f_{239} = 0.9056$ $\qquad\qquad$ $f_{240} = 0.0568$
$f_{241} = 0.0228$ $\qquad\qquad$ $f_{242} = 0.0148$

From Page 4, the atomic weight of the mixture of isotopes is

$$A_{Pu} = \sum_i (\text{atomfraction})_i * A_i$$

Substituting the values above, A = 239.1994.

2. The binding energy is given by the differences in mass between the initial and final components of the reaction. Using $A_{238} = 238.0486$ and $m_n = 1.00866$, the mass totals for ^{238}Pu neutron capture are:

$m_{238} + m_n = 239.05826$
$\qquad m_{239} = 239.0522$

$\text{binding E} = 0.00606 \text{ amu } (= 5.64 \text{ MeV})$

Similarly, for the other isotopes, the binding energies are:

^{239}Pu $\qquad$ 0.00706 amu $\qquad$ 6.58 MeV
^{240}Pu $\qquad$ 0.00566 amu $\qquad$ 5.27 MeV
^{241}Pu $\qquad$ 0.00676 amu $\qquad$ 6.30 MeV

The binding energies for ^{239}Pu and ^{241}Pu are about 1 MeV smaller than for ^{240}Pu and ^{242}Pu. ^{239}Pu and ^{241}Pu fission readily with low energy neutrons but not ^{240}Pu or ^{242}Pu.

3. The mas numbers for H, O and ^{235}U are 1,16 and 235. Inserting these values into the formula

give the atomic radii. Then, using standard geometrical relations, the area is πR^2 and the volume is $(4/3)\pi R^3$. Using the atomic weights, A, from any standard reference, the mass of each atom is given by A/N_a (see Page 3). The atomic density is then the mass divided by the volume. The following table lists the results of the calculations.

	H	O	U
Atomic Weight	1	16	235
Radius (cm)	1.250×10^{-13}	3.150×10^{-13}	7.714×10^{-13}
Area (cm^2)	4.909×10^{-26}	3.117×10^{-25}	1.869×10^{-24}
Volume (cm^3)	8.181×10^{-39}	1.309×10^{-37}	1.923×10^{-36}
Mass (g)	1.661×10^{-24}	2.657×10^{-23}	3.902×10^{-22}
Density (g/cm^3)	$2.030 \times 10^{+14}$	$2.030 \times 10^{+14}$	$2.030 \times 10^{+14}$

Note that the density is the same for each nucleus. This should be apparent when you combine the formulae for the radius and the volume of each nucleus.

4. The atomic weights of the three components of the uranium oxide are 235.0439, 238.0508 and 15.9994 (by this time it should be obvious which mass belongs to which isotope). As in Problem 1, the atomic weight of the uranium can be calculated as 235.2519 g/mole and that of the compound as 267.2507. The weight fractions of the components are then:

^{235}U	$(0.93 \times 235.2519)/267.2507 = 0.81865$
^{238}U	$(0.07 \times 235.2519)/267.2507 = 0.06162$
O	$(2 \times 15.9994)/267.2507 = 0.11973$

Then, from Page 4, the atom densities are:

^{235}U	0.02265
^{238}U	0.001684
O	0.04867

5. The atomic weights of the naturally occurring elements Fe, C, Cr and Ni are: 55.847, 12.011, 51.996 and 58.693. Calculate the atom densities used the same methods used in Problems 1 and 4. Then from Page 5, the macroscopic cross section is defined as the product of the atom density and the microscopic cross section, so the absorption and scattering Σ can be calculated directly. The total macroscopic cross section is just the sum of the two parts. All results are in the table below.

	Atom Density	Σ_a	Σ_s	Σ_t
Fe	0.060109	0.1539	0.6822	0.8361
C	0.0003153	1.1×10^{-6}	0.00149	0.00150
Cr	0.017296	0.0531	0.0585	0.1116
Ni	0.008065	0.0362	0.1436	0.1798
Steel		0.243	0.8857	1.1289